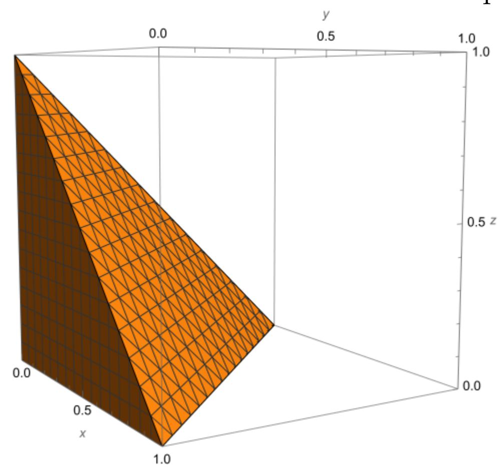
Multivariable Calculus

Quiz 10 **SOLUTIONS**

1) Compute the triple integral

$$\iiint_{\mathcal{T}} x\, dV$$

over the tetrahedron in the first quadrant bounded by the plane $x + y + z = 1$.



Solution:

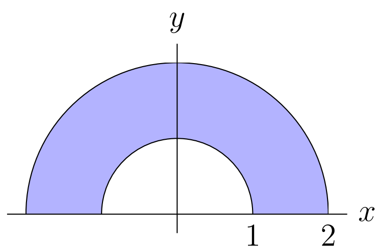
$$\begin{aligned}
\iiint_{\mathcal{T}} x\, dV &= \int_0^1 x \left[\int_0^{1-x} \left(\int_0^{1-x-y} 1\, dz \right) dy \right] dx \\
&= \int_0^1 x \left[\int_0^{1-x} (1 - x - y)\, dy \right] dx \\
&= \int_0^1 x \left[y - xy - \frac{1}{2} y^2 \right]_0^{1-x} dx \\
&= \frac{1}{2} \int_0^1 \left(x - 2x^2 + x^3 \right) dx \\
&= \frac{1}{24}
\end{aligned}$$

TURN OVER

2) Compute the double integral

$$\iint_{\mathcal{D}} y \, dA$$

by switching to polar coordinates. $\mathcal{D}$ is the half annulus between the circle of radius 2 and the circle of radius 1 in the upper half plane.



Solution: In polar, the annular region is given by $1 \le r \le 2$ and $0 \le \theta \le \pi$.

$$\begin{aligned}\iint_{\mathcal{D}} y \, dA &= \int_0^{\pi} \int_1^2 r\sin(\theta) \, r dr d\theta \\ &= \left(\int_0^{\pi} \sin(\theta)\, d\theta\right)\left(\int_1^2 r^2 \, dr\right) \\ &= [-\cos(\theta)]_0^{\pi} \left[\frac{1}{3}r^3\right]_1^2 \\ &= \frac{14}{3}\end{aligned}$$